

# Fuzzy Inventory Model for Small Scale Retailers With Local Advertisements

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**Abstract-** In this paper, we have developed an inventory model for small scale retailer with local advertisements under fuzzy environment. In reality, it is seen that we cannot define all parameters precisely due to imprecision or uncertainty in the environment. So we have defined the inventory parameters such as demand, holding cost, Initialization cost and advertising cost as trapezoidal fuzzy numbers. The signed distance method has been used for defuzzification. To illustrate the proposed model a numerical example and sensitivity analysis with respect to different associated parameter has been presented.

**Keywords:** Fuzzy inventory model, Trapezoidal Fuzzy number, Defuzzification, Advertising cost.

## 1. INTRODUCTION

The customers have wide open choice in selecting their enviable supplier. To keep hold of the consumers the retailer handles various tactics such as discount offer, concession in cost, reduction of price and also to withstand the growth of the competitors the firms are accelerated to produce distinct products bearing several attractive traits such as green products, products with eco-friendly packaging, quality improved products which are welcomed by the consumers. To make these offers fortunate to disseminate and to propagate the product, communications have to be done to the customers at the right time in the right manner. One such means is Advertising, which is becoming more important every day in this technological era.

Advertising has an extreme importance in today's society, and business companies invest constantly growing parts of their budgets in engineering ways to attract higher number of products and services they offer. (Antonio *et al.*, 2009) Advertising, a one way communication about a product or organization that is paid by a marketer to persuade an audience (readers, listeners and viewers) to a commercial offering to take some action with respect to products, ideas, or services. It is a powerful educational tool capable of reaching and motivating the customers and drives consumer behaviour for commercial purpose. Virtually any medium can be used for advertising. Commercial advertising media can include wall paintings, billboards, street furniture components, printed flyers and rack cards, magazines, newspapers, radio, cinema, television, web banners, mobile telephone, web popup. Among these mediums

billboards, handbills, television which are the traditional mediums, followed by web and email, the

recent mediums of advertising are used commonly due to its benefits and cost effectiveness.

In this paper, we convert the inventory model into fuzzy inventory model. The holding cost, demand, initialization cost, advertising cost are assumed to be the trapezoidal fuzzy numbers. For defuzzification of the total cost function, signed distance method are used. The retailer chooses handbills and television as the mode of advertising and the optimal quantity is determined for three different cases.

## 2. Definitions:

### 2.1 Trapezoidal Fuzzy Number:

The fuzzy number  $\tilde{A} = (a_1, a_2, a_3, a_4)$ , where  $a_1 < a_2 < a_3 < a_4$  and defined on  $R$  is called the trapezoidal fuzzy number, if the membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & : x < a_1 \text{ or } x > a_4 \\ \frac{(x - a_1)}{(a_2 - a_1)} & : a_1 \leq x \leq a_2 \\ 1 & : a_2 \leq x \leq a_3 \\ \frac{(x - a_4)}{(a_3 - a_4)} & : a_3 \leq x \leq a_4 \end{cases}$$

### 2.2 Signed distance method :

Let  $\tilde{A} = (a,b,c,d)$  be a trapezoidal fuzzy number, then the signed distance method of  $\tilde{A}$  is defined as  $d(\tilde{A}, 0) = \frac{1}{2} \int_0^1 [A_L(\alpha) + A_R(\alpha)]d\alpha$  where  $\tilde{A} = [A_L(\alpha), A_R(\alpha)] = [a+(b-a)\alpha, d-(d-c)\alpha]$ ,  $\alpha \in [0,1]$  is a  $\alpha$ -cut of fuzzy set  $\tilde{A}$  which is a close interval.  $d(\tilde{A}, 0) = (a + b + c + d) / 4$ .

**3. Assumptions and Notations.**

The model is developed on the following assumptions and notations.

**3.1. Assumptions**

- i) Demand rate is uniform
- ii) Lead time is zero.
- iii) the initialization cost of casting advertisement is the amount of money spent by the retailer to get in connection with the advertising agency, which is assumed to be constant irrespective of the medium.

**3.2 Notations**

- D demand
- A order cost
- c unit purchase cost
- h holding cost per unit per time
- I initialization cost
- $\Psi$  fixed cost per advertisement is telecast
- $\gamma$  variable cost per advertisement
- n number of times the advertisement is telecast
- t cost of telecasting the advertisement once
- $f_1$  fixed cost of producing a handbill.
- v variable cost of producing a handbill
- N number of employees involved in distribution of handbills
- z cost per employee
- d distance to be travelled(in km)
- a fixed cost per trip
- b variable cost per trip

**4. Mathematical Model**

**4.1. Crisp Model**

In this context, we have considered three modes of advertising cost of seller

- i) The retailer wished to advertise via local channels
- ii) To distribute hand bills to the people by hiring some persons to carry out the work.
- iii) To distribute hand bills to distant areas within the local regions.

**Case i:** The motive of advertising is to familiarize the product to the customers. A market segment consists of a large identifiable group within a market with similar wants, purchasing power, geographical location, buying attitudes. Thus the retailers telecast advertisements according to their desirable taste to make the product reach the customers. Let us consider the case where the retailer wishes to advertise via local channels. EOQ cost per cycle

$$C(Q) = A + cQ + \frac{hQ^2}{2D}$$

Advertisement cost per cycle

$$A(Q) = I + \psi + \gamma + nt$$

The total cost per unit time is  $\sigma$

$$\sigma(Q) = \frac{AD}{Q} + cD + \frac{hQ}{2} + \frac{(I + \gamma + \psi + nt)D}{Q}$$

The optimal

solution is  $Q = \sqrt{\frac{2D(A + I + \gamma + \psi + nt)}{h}}$

**Case ii :** Consider the situation where the retailer decides to advertise via handbills. He himself cannot distribute all the bills, so he hires some persons to carry out the work of distributing the hand bills to the local regions. EOQ cost per cycle  $C(Q)$

$$C(Q) = A + cQ + \frac{hQ^2}{2D}$$

Advertisement cost per cycle

$$A(Q) = I + f + v + nz$$

The total cost per unit time is  $\sigma(Q) = \frac{AD}{Q} + cD + \frac{hQ}{2} + \frac{(I + f + v + nz)D}{Q}$  The optimal

solution is  $Q = \sqrt{\frac{2D(A + I + f + v + nz)}{h}}$

**Case iii:** In this case the handbills are distributed to distant areas within the local regions.

EOQ cost per cycle  $C(Q) = A + cQ + \frac{hQ^2}{2D}$  Advertisement cost per cycle

$$A(Q) = I + f + v + nz + a + db$$

The total cost per unit time is  $\sigma(Q) = \frac{AD}{Q} + cD + \frac{hQ}{2} + \frac{(I + f + v + nz + a + db)D}{Q}$  The optimal

solution is  $Q = \sqrt{\frac{2D(A + I + f + v + nz + a + db)}{h}}$

**4.2 Fuzzy Model:** We consider the model in fuzzy environment. Due to uncertainty, it is not easy to define all parameters exactly.

**Case i:** Let  $\bar{D} = (D_1, D_2, D_3, D_4)$ ,  $\bar{h} = (h_1, h_2, h_3, h_4)$ ,  $\bar{\gamma} = (\gamma_1, \gamma_2, \gamma_3, \gamma_4)$ ,  $\bar{I} = (I_1, I_2, I_3, I_4)$ ,  $\bar{V} = (v_1, v_2, v_3, v_4)$   $\bar{b} = (b_1, b_2, b_3, b_4)$  are trapezoidal fuzzy numbers. The total variable cost per unit time

in fuzzy sense is given by  $\bar{\sigma}(Q) = \frac{AD}{Q} + cD + \frac{hQ}{2} + \frac{(\bar{I} + \bar{\gamma} + \psi + nt)\bar{D}}{Q}$  We defuzzify the

total cost  $\bar{\sigma}(Q)$  by using signed distance method

$$\sigma(Q) = \frac{1}{4}[\sigma_{ds1} + \sigma_{ds2} + \sigma_{ds3} + \sigma_{ds4}] \text{ where } \sigma_{ds1}(Q) =$$

$$\frac{AD_1}{Q} + cD_1 + \frac{h_1Q}{2} + \frac{(I_1 + \gamma_1 + \psi + nt)D_1}{Q} \quad \sigma_{ds2}(Q) =$$

$$\frac{AD_2}{Q} + cD_2 + \frac{h_2Q}{2} + \frac{(I_2 + \gamma_2 + \psi + nt)D_2}{Q} \quad \sigma_{ds3}(Q) =$$

$$\frac{AD_3}{Q} + cD_3 + \frac{h_3Q}{2} + \frac{(I_3 + \gamma_3 + \psi + nt)D_3}{Q} \quad \sigma_{ds4}(Q) =$$

$$\frac{AD_4}{Q} + cD_4 + \frac{h_4Q}{2} + \frac{(I_4 + \gamma_4 + \psi + nt)D_4}{Q}$$

To minimize total cost function per unit time, the optimal value can be obtained by solving the following equations.

$$\frac{d\sigma_{ds}(Q)}{dQ} = 0 \text{ and } \frac{d^2\sigma_{ds}(Q)}{dQ^2} > 0 \text{ then, } \frac{1}{4} \{(-1/Q^2)$$

$$[A(D_1+D_2+D_3+D_4)+I_1D_1+I_2D_2+I_3D_3+I_4D_4+\gamma_1D_1+\gamma_2D_2+\gamma_3D_3+\gamma_4D_4+(\psi+nt)(D_1+D_2+D_3+D_4)]+1/2(h_1+h_2+h_3+h_4) = 0, \text{ and } Q^* = \frac{\sqrt{2[A(D_1+D_2+D_3+D_4)+I_1D_1+I_2D_2+I_3D_3+I_4D_4+\gamma_1D_1+\gamma_2D_2+\gamma_3D_3+\gamma_4D_4+(\psi+nt)(D_1+D_2+D_3+D_4)]+h_1+h_2+h_3+h_4}}{h_1+h_2+h_3+h_4}$$

**Case ii:** Let  $\bar{D} = (D_1, D_2, D_3, D_4)$ ,  $\bar{h} = (h_1, h_2, h_3, h_4)$ ,  $\bar{\gamma} = (\gamma_1, \gamma_2, \gamma_3, \gamma_4)$ ,  $\bar{I} = (I_1, I_2, I_3, I_4)$ ,  $\bar{V} = (v_1, v_2, v_3, v_4)$   $\bar{b} = (b_1, b_2, b_3, b_4)$  are trapezoidal fuzzy numbers. The total variable cost per unit time in fuzzy sense is given by

$$\bar{\sigma}(Q) = \frac{AD}{Q} + cD + \frac{hQ}{2} + \frac{(\bar{I} + \bar{v} + f + nz)\bar{D}}{Q}$$
 We

defuzzify the total cost  $\bar{\sigma}(Q)$  by using signed

$$\text{distance method } \sigma(Q) = \frac{1}{4}[\sigma_{ds1} + \sigma_{ds2} + \sigma_{ds3} + \sigma_{ds4}]$$

where

$$\sigma_{ds1}(Q) = \frac{AD_1}{Q} + cD_1 + \frac{h_1Q}{2} + \frac{(I_1 + v_1 + f + nz)D_1}{Q} \quad \sigma_{ds2}(Q) =$$

$$\frac{AD_2}{Q} + cD_2 + \frac{h_2Q}{2} + \frac{(I_2 + v_2 + f + nz)D_2}{Q} \quad \sigma_{ds3}(Q) =$$

$$\frac{AD_3}{Q} + cD_3 + \frac{h_3Q}{2} + \frac{(I_3 + v_3 + f + nz)D_3}{Q} \quad \sigma_{ds4}(Q) =$$

$$\frac{AD_4}{Q} + cD_4 + \frac{h_4Q}{2} + \frac{(I_4 + v_4 + f + nz)D_4}{Q}$$

To minimize total cost function per unit time, the optimal value can be obtained by solving the

$$\text{following equations. } \frac{d\sigma_{ds}(Q)}{dQ} = 0 \text{ and } \frac{d^2\sigma_{ds}(Q)}{dQ^2} > 0$$

then,

$$\frac{1}{4} \{(-1/Q^2)[A(D_1+D_2+D_3+D_4)+I_1D_1+I_2D_2+I_3D_3+I_4D_4+v_1D_1+v_2D_2+v_3D_3+v_4D_4+(f+nz)(D_1+D_2+D_3+D_4)]+1/2(h_1+h_2+h_3+h_4) = 0, \text{ and } Q^* = \frac{\sqrt{2[A(D_1+D_2+D_3+D_4)+I_1D_1+I_2D_2+I_3D_3+I_4D_4+v_1D_1+v_2D_2+v_3D_3+v_4D_4+(f+nz)(D_1+D_2+D_3+D_4)]+h_1+h_2+h_3+h_4}}{h_1+h_2+h_3+h_4}$$

**Case iii:** Let  $\bar{D} = (D_1, D_2, D_3, D_4)$ ,  $\bar{h} = (h_1, h_2, h_3, h_4)$ ,  $\bar{\gamma} = (\gamma_1, \gamma_2, \gamma_3, \gamma_4)$ ,  $\bar{I} = (I_1, I_2, I_3, I_4)$ ,  $\bar{V} = (v_1, v_2, v_3, v_4)$   $\bar{b} = (b_1, b_2, b_3, b_4)$  are trapezoidal fuzzy numbers. The total variable cost per unit time in fuzzy sense is given by

$$\bar{\sigma}(Q) = \frac{AD}{Q} + cD + \frac{hQ}{2} + \frac{(\bar{I} + \bar{v} + f + nz + a + db)\bar{D}}{Q}$$

We defuzzify the total cost  $\bar{\sigma}(Q)$  by using signed

$$\text{distance method } \sigma(Q) = \frac{1}{4}[\sigma_{ds1} + \sigma_{ds2} + \sigma_{ds3} + \sigma_{ds4}]$$

where

$$\sigma_{ds1}(Q) = \frac{AD_1}{Q} + cD_1 + \frac{h_1Q}{2} + \frac{(I_1 + v_1 + f + nz + a + db_1)D_1}{Q}$$

$$\sigma_{ds2}(Q) = \frac{AD_2}{Q} + cD_2 + \frac{h_2Q}{2} + \frac{(I_2 + v_2 + f + nz + a + db_2)D_2}{Q}$$

$$\sigma_{ds3}(Q) = \frac{AD_3}{Q} + cD_3 + \frac{h_3Q}{2} + \frac{(I_3 + v_3 + f + nz + a + db_3)D_3}{Q}$$

$$\sigma_{ds4}(Q) = \frac{AD_4}{Q} + cD_4 + \frac{h_4Q}{2} + \frac{(I_4 + v_4 + f + nz + a + db_4)D_4}{Q}$$

To minimize total cost function per unit time, the optimal value can be obtained by solving the

$$\text{following equations. } \frac{d\sigma_{ds}(Q)}{dQ} = 0 \text{ and } \frac{d^2\sigma_{ds}(Q)}{dQ^2} > 0$$

$$\text{then, } \frac{1}{4} \{(-1/Q^2)[A(D_1+D_2+D_3+D_4)+I_1D_1+I_2D_2+I_3D_3+I_4D_4+v_1D_1+v_2D_2+v_3D_3+v_4D_4+d(b_1D_1+b_2D_2+b_3D_3+b_4D_4)+(f+nz+a)(D_1+D_2+D_3+D_4)]+1/2(h_1+h_2+h_3+h_4) = 0, \text{ and } Q^* = \frac{\sqrt{2[A(D_1+D_2+D_3+D_4)+I_1D_1+I_2D_2+I_3D_3+I_4D_4+v_1D_1+v_2D_2+v_3D_3+v_4D_4+d(b_1D_1+b_2D_2+b_3D_3+b_4D_4)+(f+nz+a)(D_1+D_2+D_3+D_4)]+h_1+h_2+h_3+h_4}}{h_1+h_2+h_3+h_4}$$

### 5. Numerical Illustration

We consider the following numerical values of the parameters in appropriate units to analyze the model:

#### Crisp model:

A = \$100/cycle, h = \$5/unit/cycle, D=50000units/yr, I = \$200, γ = \$150, ψ=\$250, n=10, N= 10 t = 120, z= 200, f= \$150, v=\$190, a= \$100, b= \$50, d= 20  
Km Casei:

$$Q = \sqrt{\frac{2D(A+I+\gamma+\psi+nt)}{h}} = 6164 \text{ units}$$

Case ii:

$$Q = \sqrt{\frac{2D(A+I+f+v+nz)}{h}} = 7266 \text{ units}$$

Case iii:

$$Q = \sqrt{\frac{2D(A+I+f+v+nz+a+db)}{h}} = 8648 \text{ units}$$

**Fuzzy model:**

Case i: 5917 units

Case ii: 7267 units

Case iii: 8655 units

## 6. EFFECTS OF ADVERTISEMENTS ON THE CUSTOMERS AND SOCIETY

Advertisements play a major role in creating a strong opinion about the product in the minds of the customers. Advertising will only survive and grow if it focuses on being effective. Advertisements will be effective only if it help the advertisers (retailers and manufacturers) to reach the goal. All advertisers are expecting specific results based on their stated objectives which are attained only out of proper planning, creation and execution. Effective advertisement's characteristics work on two levels. First one is, the advertisers should satisfy consumer's objectives by engaging them and delivering a relevant message. The other one is that the advertisements must achieve the advertiser's objectives. The ultimate motive of the advertisers is to make the customers to buy and keep buying their goods and services, so they make use of any one of the mediums of advertising. Therefore the advertisers must be cautious in employing the apt advertising strategies to acquire customers to make their expenditure in advertising fruitful.

## 7. SENSITIVITY ANALYSIS

The sensitivity analysis is performed for checking the effectiveness of the EOQ model for infinite production rate with respect to the advertisement cost on the optimum order quantity in fuzzy sense by using signed distance method. The seller scatters his interest in maximizing the profit and in minimizing the cost of purchasing and holding cost. The seller can expand his business circle by increasing the number of local customers with this

motive the seller looks for advertising. The cost of advertising must be minimum, but at the same time it must be effective.

## CONCLUSION:

In this paper, a fuzzy EOQ model for small scale retailers with local advertisement using Initialisation and advertising costs are studied. The optimum results of fuzzy model were defuzzified using signed distance method. This will increase the total profit. It is concluded that the value of the optimal order quantity for different cases are much sensitive with respect to the advertisement cost and so the fuzzy model permits flexibility in the system inputs.

The outcome of this research can be extended in future to the case of advertising through internet model.

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